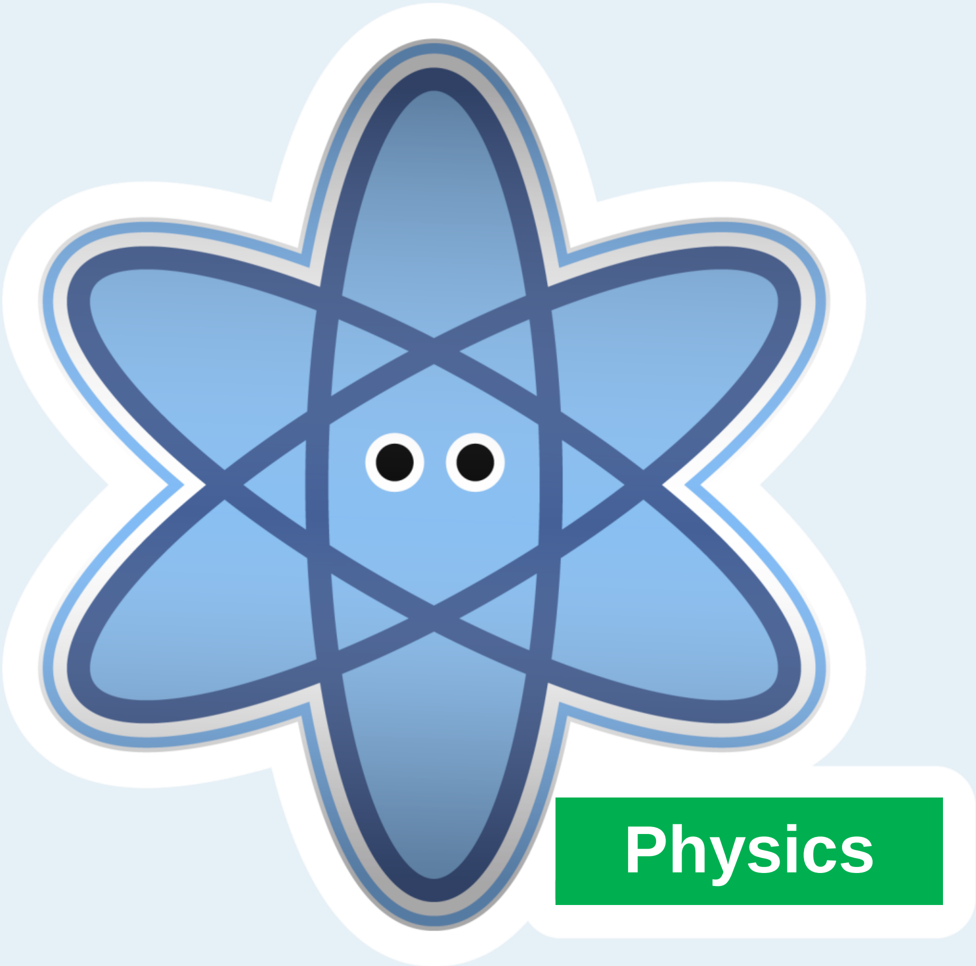

Educational Materials on the Subject

From the Earth to the Moon and Back – Gravitation in the Earth-Moon-System

FORM 11

Teacher Materials

Project Information

These educational materials have been developed in the course of the project “Columbus Eye – Live Images from the ISS in School Education”. The project Columbus Eye is sponsored by the Space Agency of the German Aerospace Centre with funds from the Federal Ministry of Economics and Energy in accordance with the resolution of the German Parliament under the funding code 50 JR 1703.

The ultimate goal of the project is to develop a wide range of digital learning materials to be used in school education, including interactive learning tools and worksheets, provided via a learning portal.

Overview

Form	11
Level	●●●○○
Time required	2 hours
Author	**Claudia Lindner**

Subjects

gravitation, elevation models, tides, Kepler’s laws, interactions, solar system, geologic history, lunar eclipse, solar eclipse

Goals

The pupils are to…

- realise the effects of differential gravitation,
- realise interactions in the earth-moon force system,
- chose physical values reasonably and deductively process them in a hypothesis,
- perform a thought experiment.

Media & Materials

- worksheet “From the Earth to the Moon and Back“
- background information regarding physics and programming
- image target „earth“
- App “The Earth-Moon-System“

link to the App: https://play.google.com/store/apps/details?id=com.ColumbusEye.Main

Didactic Commentary

Lesson Plan

Phase 0 (preparation): The app should be downloaded by the students a couple of days before the lesson. Simply share the link via e-mail or social media, project the QR code on the wall of the classroom or simply write down the link on the blackboard, so the students will be able to find the app in the Google Play Store. The app is free of charge and it has minimal legal requirements. However, due to problems with operating systems or hardware, it may not work with every single smartphone. The app is only compatible with Android phones (V5 and upwards) and it does not work on iOS yet. The image of the earth has a marker function and therefore it should be printed in good printing quality and in matt colour. Depending on the size of the classroom 2-4 copies should suffice. The copies should be put on the wall and two metres from one another. To ensure a good performance of the app, the printed images should not be bent or rolled.

Depending on the knowledge level of the students, certain tasks can be declared as optional or additional and teachers can always decide for themselves how much help or additional materials they want to give.

Phase 1: The earth image should be put on the wall in a well-lit room or outside of the building. The students need several metres of space to the back. That way two students will be able to work with each image without any problems. Small crosses on the blackboard or star stickers work very well, since the app works with edge detection. Tasks 1 and 2 are best solved in small groups, to make sure that all the students have access to a smartphone which supports the app. The tasks can also be given as homework.

Phase 2: The math problems in task 3 and the reading and discussion exercises in task 4 are best solved in single work. They are also well suited as homework.

Phase 3: Tasks 5 and 6a once again need to be solved in small groups with at least one smartphone which supports the app. The calculations in task 6b can be carried out in the same groups. This way, the students will have to discuss the exercises amongst each other and maybe they will divide the calculation work.

Model Solutions

1. The earth appears as a 3D model.
    a. With a normal A4 print we have 1m = 100.000 km, or rather 1:100.000.000. With smaller or bigger prints, the scale changes accordingly.
    b. The moon comes so close to the earth that the difference between the gravitational forces on the inside and outside becomes bigger than the gravitational forces holding the moon together. Thus, the moon is torn apart. The limit at which gravitational forces tear a moon apart is called the Roche limit. According to Kepler's laws, the side of the moon directed at the planet is moving faster than the other side. This will produce rings, much like the rings of Saturn. Mars' moon Phobos is approaching its planet slowly and will be torn apart into a ring in approx. 70 million years. Concerning the rings of Saturn, it is presumed that they partially consist of torn apart moons and that there might be an ongoing cycle of agglomerating and tearing apart of moons.
2. From a bigger distance not much happens, but the closer the moon in the smartphone gets to the image of the earth, the faster the changes in tidal range become.
    a. Graph:

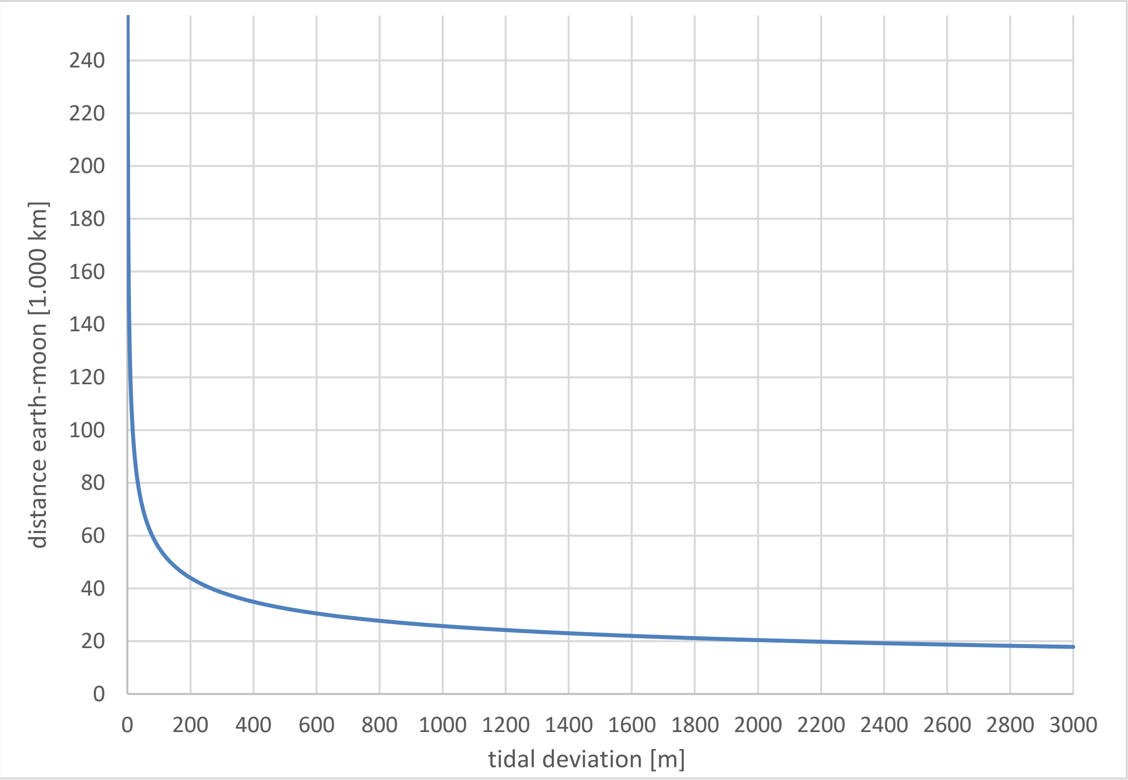

Image 1: Effect of moon distance on tidal deviation (source: own research).

The tidal energy on earth depends on the mass of the moon, the gravitational constant, the distance between earth and moon and the radius of the earth. The actual tidal range is furthermore affected by local circumstances such as ocean depth and spacious coastlines, which divert the stream of the ocean water

   b. On the moon, moonquakes occur on a regular basis, when its crust is lifted due to the gravitation of the earth. The same thing happens on the earth, but the effect is rather small compared to the increase of the water level. On moons that circle other planets like e.g. Jupiter, the gravitation is so strong that the moons are deformed. This can lead to volcanism and tectonic activity (Io) or the melting of ice crusts through friction caused by deformation (Europa).

3. Barycentre:
   a. r_{E-Min} = 4.331 km; r_{E-Max} = 4.943 km from the earth's centre or rather
      r_{M-Min} = 352.078 km; r_{M-Max} = 401.797 km from the moon's centre.
   b. Solve the formula for r: $r = r_E \cdot \frac{m_E+m_M}{m_M}$ equals r = 524.884 km
   c. Approx. 3,1 or rather 4,4 billion years. An example from our solar system: Pluto and Charon circle around a common centre of gravity 2.360 km from the centre of Pluto or rather 1.200 km from Pluto's surface. Also, the barycentre of the sun and Jupiter, containing 70% of the mass of the solar system (without the sun), lies outside the sun. The combined gravitation of all celestial bodies in the solar system also forms a barycentre.
   d. There are stars which are periodically swaying, because they are being orbited by heavy planets and revolving around the common barycentre. In the beginning scientists could only use this method to discover planets bigger and heavier than Jupiter and heavy rocky planets in other star systems.
4. Formation of the system (these scenarios do not assert a claim to be complete – the students will most likely find even more points):
   a. Extreme tides which move the still liquid lithosphere obstruct the formation of solid rock plates. The moon revolves around the young earth in less than 24 hours, which itself is turning faster. The moon appears brighter and ten times as big as today. On the dayside in low latitudes of the earth, the sun is covered for several hours every day. At the same time the nightside is intensely illuminated in the mornings and evenings and during (almost) full moon, the moon is always within the earth's shadow at night.
   b. The tides become slightly lower; due to the cubic dependency from the distance, the distance would have to increase considerably to produce a significant difference. Many organisms which are dependent on the tide (e.g. the fauna and flora in the mudflats) would have to adapt or die. Tidal power plants would become useless. Solar eclipses would become rarer and annular. Lunar eclipses would become rarer, too.
5. Through the earth's atmosphere, the moon would be magnified as if through a lens, which is why from the ground it would appear especially huge when rising or setting. The ISS orbits the earth 400 km above the ground, which is far enough outside the earth's atmosphere to see the moon's actual size. Furthermore, the human brain does not have a size comparison for the moon in the sky, which makes it appear even bigger in the human perception. For the cameras on the other hand, only the actual distance is relevant, which is why the moon always appears to be tiny on camera images if no lens-tricks are used.

6. Lunar- and solar eclipse:
    a. Producing a solar eclipse is not that easy. Beyond 200.000 km it gets especially hard to even make out the moon's shadow (it would still work best above the Sahara). The purpose of this scenario is to explain the rareness of solar eclipses and their narrow shadows.
    b. Since the sun with its diameter of 1.391.000 km emits its light all around, the shadows of the earth and the moon have a conical shape. And as the relation of the sun's radius minus the earth's radius to the earth-sun distance is the same as the relation of the earth's radius to the earth's shadow (or rather as the same is true for the distance between sun and moon), we obtain the following formula for the shadow length r_{SL}:

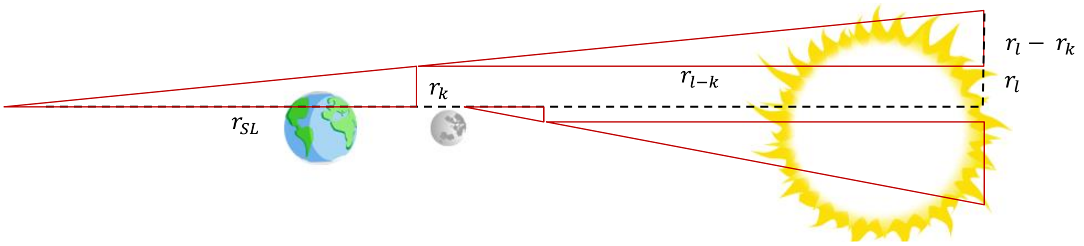

Image 2: relations of light emission and shadowing between sun and earth or rather sun and moon. (source: own illustration)

$$r_{SL} = r_k \cdot \frac{r_{l-k}}{r_l - r_k}$$

with:

r_{SL}: shadow length

r_k: radius of the body casting the shadow

r_l: radius of the body emitting the light

r_{l-k}: distance between the body emitting the light and the body casting the shadow

    c. This results in:

	At a minimal distance to the sun	At a maximum distance to the sun
Length of the earth's shadow*	1.360.000 km	1.406.000 km
Length of the moon's shadow during new moon*	367.000 km	380.000 km

*rounded to 1.000 km

The moon's shadow is longer than the minimum distance between the earth and the moon, but significantly shorter than the maximum distance. As a result, the moon's shadow does not touch the earth during every orbit when it is new moon and depending on the position of the three celestial bodies, there can only be an annular solar eclipse. At a minimal moon distance, an inclination of 5° of the moon's orbit corresponds to a deviation of 31.063 km. At an earth-diameter of 12.742 km, the moon's shadow can very well miss the earth.

The formula

$$r_S = \frac{r_E \cdot (r_{ES} - r_{E-M})}{r_{ES}}$$

can be used to calculate the radii of the earth's shadow with different sun- and moon distances. The highest figure occurs with the insertion of the maximum distance between sun and earth and with the minimum distance between earth and moon: 4.765 km. The lowest figure occurs with the minimal distance between sun and earth and with the maximum distance between earth and moon: 4.701 km. Due to the inclined orbit of the moon, it can pass significantly north or south of the earth's shadow.