



Educational Materials on the Subject

From the Earth to the Moon and Back – Gravitation in the Earth-Moon-System

FORM 11

Additional Material

Mathematical, Physical and Geographical Background Information within the App

Earth Image and Earth Model

The earth image was provided by the portal explorer.worldwind.earth/. The frame with the hexagon bolt heads is more than mere decoration: The recognition of the image as image target within the app is based on edge detection. Even the scale and the logos at the side of the page serve the purpose of stabilising the app objects within the room. The superimposed earth model has been developed by game designer Harry Edwards. It can be accessed for free.

Distance Determination

Within the app, one can find the image target, saved in a set, three-dimensional coordinate system. The position of the camera is determined based on side ratio and angles. The distance is determined based on the length, or rather the magnitude of the vector. The image target serves as reference for the measuring unit. Since the size of the image is indicated in cm, as a reference distance without measuring unit, the distance is calculated relatively in cm (provided that the sheet has been printed in A4). With regards to the earth moon distance it is equipped with the simple attachment “.000 km”.

Revolution Period / Month Length

According to the third law of Kepler, the squares of the revolution periods of two planets behave like the third powers of the large half-axes of their orbits. This is also true for moon orbits

within the same planetary system. A circular orbit is assumed, because orbit eccentricity cannot be expressed through a single smartphone position.

Moon Shadow

Due to the large radius of the sun, the real moon shadow has a conical form and various degrees of darkening. To not overburden the computing capacity of the smartphones, the app works with a one-directional light source and hard shadow edges. Therefore, the size of the shadow needs to be altered, depending on the distance. The shadow diameter is altered based on the same linear equation which was already used in task 6.

Tidal Range: Calculation

The tidal acceleration a_g on the earth's surface is calculated by:

$$a_g = \mp 2R \frac{GM}{r^3}$$

with:

R : earth radius,

G : gravitational constant,

M : mass of the moon,

r : distance between the bodies.

Here, not only the gravitation of the moon has an impact, but also the gravitation of the sun and, theoretically, the gravitation of all the other planets in the solar system. At this, the sun accounts for 14 cm which must be added or subtracted from the tide, depending on the

position of the moon. The effect of the other planets is negligible.

Based on an average deflection of 0,3 m at a medium moon distance, during the next step, we assume a linear dependence of the tidal range on the tidal acceleration. On the physics level, this is greatly simplified, but it helps the students with the work step and the calculations. Due to insufficient computing capacity, it is still impossible to calculate the changes, because they greatly depend on topography (i.e. sea depth and coastal forms). Physical models mostly assume an equally deep ocean on an evenly shaped ground.

Tidal Range: Representation

The image presentation is based on an elevation model. The land data is provided by ASTER DGEM (NASA & JAXA), a digital elevation model with an altitude resolution of 1 m, composed of radar data. The sea data is derived from bathymetric data (North Sea: Geopotential Deutsche Nordsee, Baltic Sea: Baltic Sea Bathymetry Database). The basic data has a spatial resolution of 30-250 m. On the smartphone display, however, the images have a spatial resolution of only about 3 km.

The elevation model has grey levels in 255 grades, whereby one grade corresponds to 1 m difference in height. Based on the threshold values from the tidal range calculations, it is determined which pixels of the images are overlaid with blue and which ones are not. The rendering of the ocean overlay probably is the most resource intensive calculation of the app.

The Sentinel-RGB consists of overall 9 scenes between the Dutch North Sea coast and the Danish Baltic Sea coast. The images were taken by satellite Sentinel-2A from a height of 789km above the earth's surface. The satellite has been developed by the European Space Agency and in the course of the Copernicus programme, which has already brought three couples of twin satellites with distinctive tasks and sensors into space. The resulting images can be downloaded for free under scihub.copernicus.eu/. They can be processed, using the free software SNAP (Sentinel Application Platform).

The Moon: App-Object

Here, the moon is implemented as a simple sphere onto which an equirectangular map of the moon is projected. The distortion of the equirectangular projection works with spherical coordinates, which are simply transferred into map coordinates or rather pixel coordinates. This counteracts the distortion of the map, when being transferred onto the new sphere. The moon sphere is held stable within space through a script, by being turned against the rotation of the smartphone.

Breaking Moon: Physics

Due to the cubic relation between the distance of the two bodies and their interacting forces, at some point, when the moon comes too close to the earth, it is torn apart on its front- and backside by the difference of the forces. According to the third law of Kepler, the fragments closer to the central body move faster than the more distant ones, which results in the

formation of a ring. The moon's breaking point or rather Roche limit (named after its discoverer) lies at a turning radius of between 18.261 and 9.496 km. The exact value mostly depends on the flexibility and density distribution of the moon. The app-moon starts to deform at a distance of 18.000 km; it breaks apart at 10.000 km. At this point, the moon's surface is only 3600 km away from the surface of the earth. The tidal energy would trigger kilometre high lava-tides on earth – an experiment that rather should not be tried for real.

Breaking Moon: Representation

Due to the shadow adaptation, the moon model on the screen is independent from the moon's shadow. However, both are deformed based on the same principle. Depending on the distance, the moon is stretched on the x-axis and proportionally compressed on the two other axes, once a threshold value of 18 is reached. The stretching is described on the x-axis to represent the distortion through the differences of the orbit time, even though the app-moon does not move on any orbit (unless the students simulate it). Also depending on the distance, a

different texture is superimposed onto the spherical moon model to represent the tearing apart of the moon.

HDEV-Video

In 2014 the ISS Columbus-module has been equipped with four commercially available HD video cameras, to test the durability of ordinary cameras, compared to special, more expensive ones, designed specifically for the use in space. The four cameras have fixed settings, regarding their focal length and light sensitivity and they are installed in fixed positions. Theoretically two of them are always directed backwards, one forwards and one downwards. Through turning manoeuvres of the ISS during e.g. orbit corrections, or adjustments necessary for the landing of Soyuz capsules, the cameras' angles occasionally must be changed. The cameras take turn recording a continuous video and send it back to earth, whenever the ISS is within reach of the receiving stations. One of the tasks of the Ruhr-University's "Columbus Eye"-project is to cut highlight-videos from the footage taken from this stream and archive them.